Elysium Journal of Engineering Research & Management
P-ISSN: 2347-4408
E-ISSN: 2347-4734

QUALITATIVE BEHAVIOR OF A SECOND ORDER DELAY DYNAMIC EQUATIONS

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Abstract : ***In this paper we study the qualitative behavior of delay dynamic equation of the form***

$$\left(\frac{1}{r(t)}y^{\Delta}(t)\right)^{\Delta}+p(t)y(\tau(t))=0,\ t\in T........(1)$$

using Ricatti substitution method.

1. INTRODUCTION

The theory of time scales, which has recently established a lot of attention, was introduced by Hilger in his Ph.D. Thesis in 1988 in order to combine continuous and discrete analysis. A time scale T is an arbitrary closed subset of the reals, and the cases when this time scale is equal to the reals or to the integers represent the usual theories of differential and of difference equations. Many other remarkable time scales exist, and they give rise to plenty of applications, among them the study of population dynamic models which are discrete in season and may follow a difference scheme with variable step-size or often model by continuous dynamic systems die out, say in winter, while their eggs are incubating or hidden, and then in season again, hatching gives rise to a non overlapping population .It not only unify the theories of differential equations and difference equations but also it extends these classical cases to cases “in between”, for example, to the so-called *q*-difference equations when

$$T=qN_0=\{t\in N_0\ ,\quad q>1\}$$

which has important applications in quantum theory and can be applied on various types of time scales like

T=hN, T = N_2, and T =T_n the space of the harmonic numbers.

Consider the second order delay dynamic equation(1)

$$\left(\frac{1}{r(t)}y^{\Delta}(t)\right)^{\Delta}+p(t)y(\tau(t))=0$$

Where p (t), r (t) are positive right dense continous functions defined on T and $\tau:T\to T$ satisfies $\tau(t)\le t$ for every tЄT and $\lim_{t\to\infty}\tau(t)=\infty$

A solution $y(t)$ of equation (1) is said to be oscillatory if it is neither eventually positive nor eventually negative, that is if for every $b>a$ there exists $t>b$ such that y(t)=0 or y(t)y(σ(t))<0 ; otherwise it is called nonoscillatory.Since we are interested in qualitative behavior of solutions, we will suppose that the time scale T under considerations is not bounded above and therefore the time scale is in the form $[t_0,\infty)_T=[t_0,\infty)\cap T$.

Note that if T=N we have

$$\sigma(n)=n+1$$
$$\mu(n)=1$$
$$y^{\Delta}(n)=\Delta y(n)$$

then (1) becomes,

$$\Delta\left(\frac{1}{r(n)}\Delta y(n)\right)+p(n)y(\tau(n))=0\quad ,n\in N$$

If T=R we have

$$\sigma(t)=t$$
$$\mu(t)=0$$
$$f^{\Delta}(t)=f'(t).$$

then equation (1.) becomes

$$\left(\frac{1}{r(t)}y'(t)\right)'+p(t)y(\tau(t))=0$$

If $T=hN$, $h>0$, we have

$$\sigma(t)=t+h$$
$$\mu(t)=h$$
$$y^{\Delta}(t)=\Delta_h(t)=\frac{y(t+h)-y(t)}{h}$$

then equation (1) becomes

$$\Delta_h\left(\frac{1}{r(t)}\Delta_h(y(t))\right)+p(t)y(\tau(t))=0$$

MAIN RESULT

Theorem 1.

Assume that

$$(i)\ M(t)=\int_{t_0}^{t}r(s)\Delta s\to\infty\ \ as\ t\to\infty$$

$$(ii)\ \tau^{\Delta}(t)\ge 0$$

$$(iii)\int^{\infty}\left(M(\tau(t)p(t)-\frac{r(\tau(t))\tau^{\Delta}(t)}{4M(\tau(t))}\right)\Delta t=\infty$$ Then

(1) is oscillatory.

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Proof

Let y(t) be a non oscillatory solution of (1). Without loss of generality we may assume that y(t)>0 and y($\tau(t)>0$ for $t\geq t_1$

From (1) we have $\left(\frac{1}{r(t)}y^{\Delta}(t)\right)^{\Delta}<0$

This implies that(from (2)) $\left(\frac{1}{r(t)}y^{\Delta}(t)\right)>0$

Now make the Ricatti substitution

$$V(t)=\frac{M(\tau(t))}{y(\tau(t))}\left(\frac{y^{\Delta}(t)}{r(t)}\right)$$

Then V (t)>0.Now

$$V^{\Delta}(t)=\left(\frac{M(\tau(t))}{y(\tau(t))}\right)\left(\frac{y^{\Delta}(t)}{r(t)}\right)^{\Delta}$$

$$+\left(\frac{M(\tau(t))}{y(\tau(t))}\right)^{\Delta}\left(\frac{y^{\Delta}(t)}{r(t)}\right)^{\sigma}$$

$$=\left(\frac{M(\tau(t))}{y(\tau(t))}\right)\left(\frac{y^{\Delta}(t)}{r(t)}\right)^{\Delta}$$

$$+\left(\frac{y^{\Delta}(t)}{r(t)}\right)^{\sigma}\left(\frac{y(\tau(t))r(\tau(t))\tau^{\Delta}(t)-M(\tau(t))y^{\Delta}(\tau(t))\tau^{\Delta}(t)}{y(\tau(t))y^{\sigma}(\tau(t))}\right)$$

$$=-M(\tau(t))p(t)+\frac{r(\tau(t)\tau^{\Delta}(t)}{M(\tau(t)}V^{\sigma}(t)$$

$$-V^{\sigma}(t)\frac{y^{\Delta}(\tau(t)}{r(\tau(t)}\times\frac{r(\tau(t)\tau^{\Delta}(t)}{y(\tau(t)}$$

Since $\left(\frac{y^{\Delta}(t)}{r(t)}\right)$ is decreasing , $\frac{y^{\Delta}(\tau(t)}{r(\tau(t)}\geq\frac{y^{\Delta}(t)}{r(t)}$

$$\therefore V^{\Delta}(t)\leq\frac{r(\tau(t))\tau^{\Delta}(t)}{M(\tau(t))}V^{\sigma}(t)-\frac{r(\tau(t))\tau^{\Delta}(t)}{y(\tau(t))}\frac{y^{\Delta}(t)}{r(t)}V^{\sigma}(t)$$

$$-M(\tau(t))p(t).$$

$$=\frac{r(\tau(t))\tau^{\Delta}(t)}{M(\tau(t))}\left(V^{\sigma}(t)-(V^{\sigma}(t)^2\right)$$

$$-M(\tau(t))p(t)$$

As the polynomial $P(V)=V^{\sigma}-(V^{\sigma})^2\leq\frac{1}{4}$

$$V^{\Delta}(t)\leq\frac{r(\tau(t))\tau^{\Delta}(t)}{4M(\tau(t)}-M(\tau(t))p(t)$$

Integrating from t_1 to t we get

$$V(t)\leq V(t_1)-\int_{t_1}^{t}\left(M(\tau(s))p(s)-\frac{r(\tau(s)\tau^{\Delta}(s)}{4M(\tau(s))}\right)\Delta s$$

Letting $t\rightarrow\infty$ V(t) $\rightarrow-\infty$

Which is a contradiction and hence the result.

Corollary

Assume that (i) and (ii) are satisfied and if

$$\liminf_{t\rightarrow\infty}\frac{M^2(t)p(t)}{r(t)}>\frac{1}{4}$$

then the equation $\left(\frac{1}{r(t)}y^{\Delta}(t)\right)^{\Delta}+p(t)y(t)=0$

is oscillatory.

Proof

Put $\tau(t)=t$ in

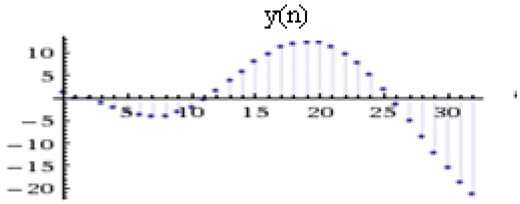
$$\int^{\infty}\left(M(\tau(t)p(t)-\frac{r(\tau(t))\tau^{\Delta}(t)}{4M(\tau(t))}\right)\Delta t=\infty$$

We get the result.

Example

Consider the dynamic equation of the form

$$\left(\frac{1}{t}\right)(y(t))^{\Delta\Delta}+\frac{1}{t^2}y(t-1)=0,\ t\in[1,\infty)_T \text{ ....(2)}$$



Hence every solution of equation (2) is oscillatory.

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